**Motivating Question**: what should the likelihood look like for the state-space model, given that the process errors can be considered parameters (after all, JAGS estimates them!)

For the state space model, we have a state equation where n(t) is a noise process:

and an observation equation where v(t) is a measurement noise process.

In the bird model, at each timestep we calculate the state equation:

**State Equation**

; the usual population bookkeeping (detail not shown), where Nm is the hidden process, i.e. the number of mature birds alive.

In the MCMC search, lognormal process error is added by drawing from a normal distribution with .

…and the estimated colony count (mature birds that don't skip breeding) is a function of *x*

#SK is the skipping rate

**Process error** is:

#(x is on the log scale, Nm is not, so this is the difference of two logs);

**Observation error** is:

(where y (observed colony count) is a log and CC is not; so again this is the difference of two logs)

My best guess at the likelihood is that it should be:

where n = 64 years in this particular data set.

Given that the errors are already on the log scale:

And the AIC would be:

where , i.e., the 15 or so parameters we care about, plus 64 \* 2 parameters for the process and observation errors, plus the 2 standard deviations (for process and observation errors).

However, the way parameters, p, are counted is somewhat mystifying. Carl Walters wrote:

*“John,*

*That likelihood function looks right. Note for model comparisons you can drop the two-pi constant terms,and if Jags is estimating the two variances you can also drop the positive ratio terms to just leave the kernel nlog(sigmaproc)+nlog(sigmaobs). The ML estimates of the variances are basically just the sums of squares deviations divided by number of observations.  So evaluated at these ML estimates, the ss/var ratios  just become ss/(ss/n)=1/n, which can be ignored in likelihood comparisons.*

*There is an issue about the effective number of parameters being estimated in random effects models.  I think it should be the total count that you identified, but others use a lower count for the random parameters.  Google “AIC for mixed models” for papers about approximate number of effective parameters, which is likely in your case to just be around 15+2+2, i.e. only two parameters in the count for random effects.  I frankly do not understand the derivations that lead to this and related estimators.  One point that may help is that absent observation errors, all the process error values needed to exactly fit the data are in fact implied by the 15 structural parameters, so those process errors should not be treated as additional parameters at all…”*

Therefore, what you’ll see in the revised model is:

where the number of parameters is calculated as follows:

= the number of covariates used in the model for each of the four main parameters, *reproductive productivity, survival, skipping rate, and reproductive capacity*.

The full set of possible covariates is: 1) an El Niño index (either EN\_ICEN or ULOMZ), 2) anchoveta biomass, and 3) fishing mortality, F. Each of the main parameters also has an intercept term which is the average rate, so we add 1 to the count for each:

where:

* is reproduction
* is survival
* is skipping
* is reproductive carrying capacity, which only depends only on anchoveta

plus

* 1 for n0 (initial population size)
* 2 for standard deviations of process and observation errors.

In most of the runs I did, p = 17.